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Sample Allocation Procedure for Assessment of Impact of Development Programme under Joint effect of Linear and Exponential Phases

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Abstract— Verma et al. (2014a) suggested sample allocation method for the assessment of the impact of the development programmes when the effect of phase is linear in nature. Verma et al. (2014b) discussed sample allocation procedure method for the exponential phase effect. However the effect of the phases can varied between linear as well as exponential in real life situations. So in the present paper we have proposed general sample allocation methods which assumed that the impacts of the phases are proportional to linear and exponential phase effects. The proposed sample allocation procedures are general sample allocation methods because proportional, optimum, Neyman, Verma et al. (2014a) and Verma et al. (2014b) sample allocation procedures are the particular cases of the proposed allocation procedures. The paper concludes with empirical study..

Keywords— *Sample Allocation; Optimum Allocation; Neyman Allocation.*

1. Introduction

Sample allocation is a very important aspect in stratified sampling. The researchers have to decide about the sample allocation to be followed in different strata before drawing the sample. Many sample allocation methods are available in literature such as equal, proportional and optimum allocations. The allocation selected depends on the nature of population under study (Ahsan et al 2005). Since the developmental programmes are implemented in phased manner hence the units of different strata/ phases receive different impacts. Hence usual sample allocation methods cannot be applied as these methods do not account for the actual imposed variability in different phases. However the heterogeneity in the population based on the nature of impact may be used for sample selection from different strata by assigning weights (Pandey and Verma, 2008). Different development programmes such as health schemes, literacy drive and poverty alleviation programmes are usually implemented in large geographical areas in different phases. So in such situations sample allocation depends not only the number of units in different phases/strata but also on the impact of the development programme. Hence the general sample allocation procedures can not be applied as the time factor also play an important role in implementation of the development programme and sample selection thereof (Verma et al. 2012) . In the literature different allocation methods for development programmes such as additive impact (Pandey and Verma, 2008), multiplicative impact (Pandey, 2010), joint impact of multiplicative and additive impact (Pandey, 2011), linear impact (Verma et al. 2012), exponential impact (Verma et al. 2014b) already have been discussed. All these sample allocation methods assume that costs of observing the units in different phases are same. But in general costs of sample selection can not be same in all phases. Hence Verma et al. (2014a) suggested a sample allocation method for studying the impact of development programmes under linear phase impact for the variable cost in different phases. Verma and Pandey (2015) proposed sample allocation procedures under geometric phase impact under cost constraints. So we have suggested sample allocation procedures which can be implemented when the impacts of phases are proportional to linear and exponential phase effect for variable cost of sample selection.

2. Sample Allocation Procedures under Cost Constraints

We assume that a development programme is being implemented in different phases for different population units. Suppose $N_1, N_2, N_3.....N_h$ are the beneficiaries of the development programme implemented in different phases. Let the impact of the development programme is proportional to the product of linear and exponential trend in different phases. If there are L phases and the impact of last phase (the most recent one) is [a] then the impact of just preceded phase will be $[a+b]e^d$. Therefore for the first year if there are N_1 units, then the actual impact per unit will be $[a+b(L-1)]e^{d(L-1)}$ as compared to the last phase. Hence, for comparison with the last units, the total impact of the first phase units will be $[a+b(L-1)]e^{d(L-1)}N_1$ as compared to last phase units impact aN_L. In the second phase development programme will provide benefit to N_2 beneficiaries. Hence total impact of the units will be $[a+b(L-2)]e^{d(L-2)}N_2$. Similarly the total impact for the third phase will be $[a+b(L-3)]e^{d(L-3)}N_3$. In general total impact of the h-th phase units will be $[a+b(L-h)]e^{d(L-h)}N_h$. The complete procedure has been represented in Table 1 for more clarity.

We cannot apply proportional allocation for allocating the samples to strata for the impact evaluation of development programme, since it does not take into account the temporal variation due to phase implementation of the development programme (Pandey and Verma, 2008). The proportional allocation is appropriate when different parts of population are proportionally represented in the sample (Buddhakusomsiri and Parthanadee, 2008). But in the proposed sample allocation methods number of units selected from each stratum depends directly on the number of units in the stratum as well as the effect of the phases.

2.1 Procedure-I

The number of units selected from h-th stratum directly depends on the number of units in the h-th stratum.

$$n_h \propto N_h \tag{1}$$

Because the development programmes are being implemented in phased manner and number of units selected from different phases will also depends on the temporal effect of phases. When the impact of phases is linear then the number of units selected in h-th phase will depends on the phased effect of the h-th unit. Therefore, we can write

$$n_h \propto [a+b(L-h)] \tag{2}$$

When the impact in different phases follows exponential trend then the number of units selected in h-th phase will depends on the phased effect of the h-th stratum. So we can write

$$n_h \propto [ae^{d(L-h)}] \tag{3}$$

When the impact of the phases proportional to linear and exponential phases then from equations (1), (2) and (3) we get

$$n_h \propto [a+b(L-h)][ae^{d(L-h)}]N_h \tag{4}$$

$$n_h = k[a+b(L-h)][ae^{d(L-h)}]N_h \tag{5}$$

The equation (5) can be used for allocation of sample sizes to different phases when the costs of observing the units are same. However in real life condtions costs of observing the units vary from phase to phase. Because all the development programmes are not implemented at the same time but are implemented in different in different geographical locations. Hence the cost of observing the units will vary with time or phases. Hence we have considered the following linear cost function.

$$C = C_0 + \sum_{h=1}^{L} c_h n_h \tag{6}$$

Where C= Total cost of the survey

C_0 = Overhead cost

c_h = Cost of observing one unit

Now multiplying equation (5) by c_h we get

$$c_h n_h = k[a+b(L-h)][ae^{d(L-h)}]c_h N_h \tag{7}$$

Taking the summation on both sides in (7) we get

$$\sum_{h=1}^{L} c_h n_h = k[a+b(L-h)][ae^{d(L-h)}]c_h N_h \tag{8}$$

Using equation (6) we get

$$C - C_0 = k\sum_{h=1}^{L}[a+b(L-h)][ae^{d(L-h)}]c_h N_h \tag{9}$$

$$k = \frac{C - C_0}{a\sum_{h=1}^{L}[a+b(L-h)]e^{d(L-h)}c_h N_h} \tag{10}$$

Now putting the value of k in equation (7) we get

$$n_h = \frac{(C-C_0)N_h[a+b(L-h)]e^{d(L-h)}}{\sum_{h=1}^{L} c_h N_h[a+b(L-h)]e^{d(L-h)}}$$

- The proposed sample allocation method is general allocation method for the cases when the impact of the phases is proportional to the product of linear and exponential phase effect and cost of observing the units from the population varies from phase to phase.
- Proportional allocation is also a particular case of the proposed allocation method when the joint phase effect is unity and cost of observing the units is same.
- Verma et al. (2014a) sample allocation procedure is also a particular case of the proposed sample allocation procedure when exponential phase impact is unity.
- Verma et al. (2014b) sample allocation procedure is also a particular case of the proposed sample allocation procedure when linear phase impact is unity.

2.2 Procedure-II

We know that the number of units selected from each stratum directly depends on the stratum weight and stratum standard deviation. Mathematically we can write that

$$n_h \propto W_h S_h \qquad (12)$$

The number of sample units selected from each stratum is inversely proportional to square root of the cost in the h-th stratum

$$n_h \propto \frac{1}{\sqrt{C_h}} \qquad (13)$$

As development programmes are being implemented in phased manner and number of units selected will also depends on the temporal effect of phases and we have assumed that the impact in different phases is proportional to the product of linear and exponential trend. Hence the numbers of units selected in h-th phase are given by

$$n_h \propto [a + b(L-h)][ae^{d(L-h)}] \qquad (14)$$

From equations (12), (13) and (14) we will get

$$n_h \propto \frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)][ae^{d(L-h)}] \qquad (15)$$

$$n_h = k\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)][ae^{d(L-h)}] \qquad (16)$$

Taking summation on both sides of equation (16) we get

$$\sum_{h=1}^{L} n_h = k\sum_{h=1}^{L}\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)][ae^{d(L-h)}] \qquad (17)$$

$$n = k\sum_{h=1}^{L}\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)][ae^{d(L-h)}] \qquad (18)$$

$$k = \frac{n}{\sum_{h=1}^{L}\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)][ae^{d(L-h)}]} \qquad (19)$$

Putting the value of k from equation (19) in equation (16) we get

$$n_h = \frac{\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)]e^{d(L-h)}}{\sum_{h=1}^{L}\frac{W_h S_h}{\sqrt{C_h}}[a + b(L-h)]e^{d(L-h)}} \times n \qquad (20)$$

The complete procedure has been represented in Table 2 for more clarity.

- The proposed sample allocation method is general allocation method for the cases when the impact of the phases is proportional to the product of linear and exponential phase effect and stratum weight and stratum standard deviation are known and cost of the sample selection varies from phase to phase.
- The proposed sample allocation procedure reduces to Optimum Allocation when the product of linear and exponential phase effect is unity.
- The proposed sample allocation procedure reduces to Neyman Allocation when the product of linear and exponential is effect same in all the phases and cost of observing the units in each phase is same.
- Verma et al. (2014a) sample allocation procedure is a meticulous case of the projected allocation procedure when stratum standard deviations are same and the cost of observing the each unit is same in all strata and impact of exponential phase is unity.
- *Verma et al. (2014b)* sample allocation procedure is a particular case of the proposed allocation procedure when stratum standard deviations are same and the cost of observing the each unit is same in all strata and impact of linear phase is unity.
- *Proportional allocation* is also a particular case of the proposed allocation method when the product of linear and exponential phases is unity and standard deviation in each phase is constant i.e. $S_h = \text{constant}$ and the cost of observing the units is same.

3. Empirical Study

We assume that a development programme is being implemented in seven phases (seven years) in different villages. The numbers of beneficiaries from development programme in phase 1 to 7 are given below. Suppose the cost of observing the each units are 1, 1, 1.5, 1.5, 2, 2, 2.5 units respectively in phases 1 to 7. Further we assume that $C - C_0 = 280$ units. The allocation of samples in different phases using proposed allocation procedures are given in Table 3 to Table 8 for different values of a , b and d.

4. Conclusion

We have suggested a sample allocation procedure for assessment of the development programmes when the development programmes are usually implemented in phased manner and impact of the development programmes is proportional to the product of linear and exponential trend and cost of observing the units varies from phase to phase. This procedure is general sample allocation procedure as the sample allocation procedures such as optimum, Neyman, proportional allocation, Verma et al. (2014a) and Verma et al. (2014b) sample allocation procedures are the particular cases of the proposed allocation methods. The present sample allocation procedure is best suitable for assessment of the development programmes when the population units in different phases are geographically far situated.

References

[1] Ahsan, M.J., Najmussehar and Khan, M.G. M., Mixed allocation in stratified sampling. Aligarh Journal of Statistics, 25:87-97, 2005.

[2] Buddhakulsomsiri, Jirachai and Parthanadee, Parthana, Stratified random sampling for estimating billing accuracy in health care systems, Health Care Management Science, 11:41-54, 2008.

[3] Pandey, R. , Samples allocation for program evaluation: a case of multiplicative effect on units due to developmental initiatives in phased manner, Folia Forestalia Polonica, S. A, 52(2): 83-88, 2010.

[4] Pandey, R., Samples allocation for evaluation of developmental program: A case of combined additive and multipli. impact on units of different phases. Rev. Bras. Biom, São Paulo, 29(1): 14-24, 2011.

[5] Pandey, R. and Verma, M. R., Samples allocation in different strata for impact evaluation of developmental programme. Rev. Mat. Estat . 26(4):103-112, 2008.

[6] Verma, M.R., Pandey, R., Singh, B. and Prasad, S., A methodology for determination of sample size for evaluation of development programme. International Journal of Agricultural and Statistical Sciences, 8(2), 627-632, 2012.

[7] Verma, M.R., Pandey, R. and Singh, B. (2014a). Sample Allocation Procedure for Assessment of Impact of Developmental Programmes Under Linear Phase Effect. International Journal of Agricultural and Statistical Sciences, 10(1), 225-229, 2014a.

[8] Verma, M.R, Singh, B., Pandey, Rajiv and Joorel, J.P.S., Sample allocation procedure for evaluation of the impact of development programme under exponential phase effect. Journal of the Indian Statistical Association, 52 (1), 151-160, 2014b.

[9] Verma, M.R. and Pandey, R., Sample allocation procedures for assessment of development programme under cost constraints. . International Journal of Agricultural and Statistical Sciences, 11 (sup.1), 287-292, 2015.

Table 1: Proposed Methodology -I - Steps for Determination of Sample Size

Stratum number (phase)	**Stratum size**	**Impact of Programme**	**Sample size**
1	N_1	$[a+b(L-1)]e^{d(L-1)}$	$n_1 = \frac{(C-C_0)N_1[a+b(L-1)]e^{d(L-1)}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
2	N_2	$[a+b(L-2)]e^{d(L-2)}$	$n_2 = \frac{(C-C_0)N_2[a+b(L-2)]e^{d(L-2)}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
3	N_3	$[a+b(L-3)]e^{d(L-3)}$	$n_3 = \frac{(C-C_0)N_3[a+b(L-3)]e^{d(L-3)}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
….	….	…..	….
….	….	…..	….
h	N_h	$[a+b(L-h)]e^{d(L-h)}$	$n_h = \frac{(C-C_0)N_h[a+b(L-h)]e^{d(L-h)}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
….	….	…..	….
….	….	…..	….
$L-2$	N_{L-2}	$[a+2b]e^{2d}$	$n_{L-2} = \frac{(C-C_0)N_{L-2}[a+2b]e^{2d}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
$L-1$	N_{L-1}	$[a+b]e^{d}$	$n_{L-1} = \frac{(C-C_0)N_{L-1}[a+b]e^{d}}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$
L	N_L	a	$n_L = \frac{(C-C_0)N_L a}{\sum_{h=1}^{L} c_h N_h [a+b(L-h)]e^{d(L-h)}}$