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Separate Regression-type Estimators (Generalized Class) for the Estimation of Finite Population Mean under Stratified Random Sampling

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Abstract— In this paper we proposed generalized class of separate regression-type estimators under stratified random sampling to estimate the finite population mean. Bias and Mean square error of the proposed estimator are obtained. Further study has been done to obtain an optimum class of estimators also with minimum mean square error. A class of estimators depending upon the estimated optimum value based on sample observations is also obtained to enhance the practical utility of the optimum estimator. Further comparative study has been done with some earlier estimators.

Keywords— *Auxiliary variable ; Regression estimator; Bias; Mean Square Error; Stratified Random Sampling*

1. Introduction

Stratified random sampling is used to improve the precision of estimator when population is heterogeneous. Stratification is the process of dividing members of the population into homogeneous subgroups before doing actual sampling. The strata are made mutually exclusive: such that every element in the population is assigned to only one stratum and is also exhaustive. No population element is excluded. This improves the representativeness of the sample by reducing sampling error. It produces a weighted mean that has less variability than the arithmetic mean of a simple random sample of the population. And then a sample is drawn from each stratum by simple random sampling without replacement according to definite allocation plan. The use of auxiliary variable x when it is correlated with the study variable y further increases the precision of the estimator.

We assume that the population consists of N units, which can be partitioned into L strata of sizes N1, N2….NL such that $\sum_{h=1}^{L} N_h = N$. Let (Y_{hi}, X_{hi}); (i=1,2…...Nh) denote the values of the variates (y, x) respectively for the ith unit in hth stratum and $\overline{Y}_h$ and $\overline{X}_h$ denote strata means. The strata weights are $W_h = \frac{N_h}{N}, (h = 1,2 \ldots\ldots. L)$.

Further let,

$\overline{Y} = \sum_{h=1}^{L} W_h \overline{Y}_h$ (Population mean of the study variable y)

$\overline{X} = \sum_{h=1}^{L} W_h \overline{X}_h$ (Population mean of the auxiliary variable x)

$$S_{hy}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(Y_{hi} - \overline{Y}_h\right)^2$$

$$S_{hx}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(X_{hi} - \overline{X}_h\right)^2$$

$$S_{hxy} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(X_{hi} - \overline{X}_h\right)\left(Y_{hi} - \overline{Y}_h\right)$$

$$\mu_{rs} = \frac{1}{N_h} \sum_{i=1}^{N_h} \left(X_{hi} - \overline{X}_h\right)^r \left(Y_{hi} - \overline{Y}_h\right)^s$$

$$\rho_h = \frac{S_{hxy}}{S_{hx} S_{hy}}, \qquad \beta_{2(hx)} = \frac{\mu_{40(hx)}}{\mu_{20(hx)}^2},$$

$$B_h = \frac{S_{hxy}}{S_{hx}^2} = \rho_h \frac{S_{hy}}{S_{hx}}$$

A simple random sample of size n is drawn without replacement under proportional allocation from each of the L strata i.e. $\underline{n} = (n_1, n_2 \ldots.. n_L)$, n_h denoting the number of units in the sample is drawn from the hth stratum , such that $n_h = \frac{n}{N} N_h$ and $\sum_{h=1}^{L} n_h = n$. Let, the means of the study variable y and auxiliary variable x of the n_h sample units drawn from the hth stratum whose size N_h is assumed to be known are $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ respectively. Also let

$$s_{hy}^2 = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}\left(y_{hi} - \bar{y}_h\right)^2$$

$$s_{hx}^2 = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)^2$$

$$s_{hxy} = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)\left(y_{hi} - \bar{y}_h\right)$$

$$b_h = \frac{s_{hxy}}{s_{hx}^2}$$

The proposed generalized class of separate regression-type estimator under stratified random sampling and using auxiliary variate x for the estimation of population mean of the study variate y is

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h \bar{y}_{hg} \qquad (1.1)$$

where

$$\bar{y}_{hg} = \left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{g(u)\right\} \qquad (1.2)$$

where $u = \frac{s_{hx}^2}{S_{hx}^2}$ and $g(u)$ is a bounded function of u, having first three derivatives with respect to u to be bounded and continuous such that validity conditions of Taylor's series expansion are satisfied and $g(1) = 1$.

Theorem 1.1: Bias of the proposed estimator $\bar{y}_{sg}$ is given as follows:

$$Bias\left(\bar{y}_{sg}\right) = \sum_{h=1}^{L}\frac{B_h \mu_{30(hx)}}{nS_{hx}^2}\{1 - g'(1)\} + \sum_{h=1}^{L}\frac{\mu_{21(hx)}}{n}\left(\frac{1}{S_{hx}^2} - \frac{B_h}{S_{hxy}}\right) + \sum_{h=1}^{L}\frac{\bar{Y}_h}{2!\,n}\left(\beta_{2(hx)} - 1\right)g'(1)$$

Proof: Using equation (1.2) and expanding $g(u)$ about the point $u = 1$ in the third order Taylor's series expansion,

$$\bar{y}_{hg} = \left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{g(1) + (u-1)g'(1) + \frac{(u-1)^2}{2!}g''(1) + \frac{(u-1)^3}{3!}g'''(u^*)\right\} \qquad (1.3)$$

where, $u^* = 1 + \theta(u - 1)$, $0 < \theta < 1$ and θ may depend on u . $g'(1)$, $g''(1)$ and $g'''(u^*)$ denote the first, second and third partial derivatives of $g(u)$ at the point $u = 1$, 1 and u^*, respectively.

Further let,

$\bar{y}_h = \bar{Y}_h(1 + e_0)$, $\bar{x}_h = \bar{X}_h(1 + e_1)$, $s_{hx}^2 = S_{hx}^2(1 + e_2)$, $s_{hxy} = S_{hxy}(1 + e_3)$

so that $E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0$. Also we have,

$$b_h = \frac{s_{hxy}}{s_{hx}^2} = \frac{S_{hxy}(1 + e_3)}{S_{hx}^2(1 + e_2)} = B_h\left(1 - e_2 + e_3 + e_2^2 - e_2 e_3 + ...\right) \qquad (1.4)$$

using equation (8.1.4) in (8.1.3) and writing $\bar{y}_{hg}$ in terms of ei's, we have

$$\bar{y}_{hg} = \left[\bar{Y}_h(1 + e_0) + B_h(1 - e_2 + e_3 + ...)\left\{\bar{X}_h - \bar{X}_h(1 + e_1)\right\}\right] * \left[g(1) + e_2 g'(1) + \frac{e_2^2}{2!}g''(1) + \frac{e_2^3}{3!}g'''(u^*)\right]$$

$$= \left\{\bar{Y}_h(1 + e_0) - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + ...)\right\} * \left[1 + e_2 g'(1) + \frac{e_2^2}{2!}g''(1) + ...\right]$$

$$= \bar{Y}_h(1 + e_0) - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + ...) + \bar{Y}_h(1 + e_0)e_2 g'(1) + \bar{Y}_h(1 + e_0)\frac{e_2^2}{2!}g''(1) +$$
$$B_h\bar{X}_h(-e_1 + e_1e_2 - e_1e_3 + ...)e_2 g'(1) + B_h\bar{X}_h(-e_1 + e_1e_2 - e_1e_3 + ...)\frac{e_2^2}{2!}g''(1)$$

$$= \bar{Y}_h(1 + e_0) - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + ...) + \bar{Y}_h(e_2 + e_0e_2)g'(1) + \bar{Y}_h\frac{(e_2^2 + e_0e_2^2)}{2!}g''(1) +$$
$$B_h\bar{X}_h(-e_1e_2 + ...)g'(1) + ... \qquad (1.5)$$

Using (1.5) in (1.1), we have

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\bar{Y}_h(1 + e_0) - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + ...) + \bar{Y}_h(e_2 + e_0e_2)g'(1) + \bar{Y}_h\frac{(e_2^2 + e_0e_2^2)}{2!}g''(1) + B_h\bar{X}_h(-e_1e_2 + ...)g'(1) + ...\right] \qquad (1.6)$$

Considering the terms up to $O\left(\frac{1}{n}\right)$ and taking expectation on both sides, we get $Bias\left(\bar{y}_{sg}\right) = E\left(\bar{y}_{sg} - \bar{Y}\right)$

$$= \sum_{h=1}^{L} W_h\left[\bar{Y}_h E(e_0) - B_h\bar{X}_h\{E(e_1) - E(e_1e_2) + E(e_1e_3)\} + \bar{Y}_h\{E(e_2) + E(e_0e_2)\}g'(1) + \bar{Y}_h\frac{E(e_2^2)}{2!}g''(1) - B_h\bar{X}_h E(e_1e_2)g'(1)\right]$$

(1.7)

Now using the following expressions for a simple random sample of size n_h drawn under proportion allocation i.e. $n_h = \frac{n}{N}N_h$ from each stratum of size N_h. But here we have assumed that the size of the hth stratum N_h is very large as compared to the sample size n_h of the stratum, so ignore the finite population correction term $f = \frac{n_h}{N_h}$.

$E(e_0^2) = \frac{S_{hy}^2}{n_h\bar{Y}_h^2}$, $E(e_1^2) = \frac{S_{hx}^2}{n_h\bar{X}_h^2}$,

$E(e_2^2) = \frac{1}{n_h}\left(\beta_{2(hx)} - 1\right)$, $E(e_0e_1) = \frac{S_{hxy}}{n_h\bar{X}_h\bar{Y}_h}$

$$E(e_0 e_2) = \frac{\mu_{21(hx)}}{n_h S_{hx}^2 \bar{Y}_h}\ , \qquad E(e_1 e_2) = \frac{\mu_{30(hx)}}{n_h S_{hx}^2 \bar{X}_h}\ ,$$

$$E(e_1 e_3) = \frac{\mu_{21(hx)}}{n_h S_{hxy} \bar{X}_h}$$

Now from equation (1.7), we have

$$Bias(\bar{y}_{sg}) = \sum_{h=1}^{L} \frac{B_h \mu_{30(hx)}}{nS_{hx}^2}\{1-g'(1)\} + \sum_{h=1}^{L} \frac{\mu_{21(hx)}}{n}\left(\frac{1}{S_{hx}^2} - \frac{B_h}{S_{hxy}}\right) + \sum_{h=1}^{L} \frac{\bar{Y}_h}{2!n}\left(\beta_{2(hx)} - 1\right) g''(1) \qquad (1.8)$$

Theorem 1.2: Mean square error (MSE) of estimator $\bar{y}_{sg}$, to the first of approximation is given by

$$MSE(\bar{y}_{sg}) = \sum_{h=1}^{L} W_h \frac{(1-\rho_h^2)}{n} S_{hy}^2 + \sum_{h=1}^{L} \frac{W_h \bar{Y}_h^2}{n}\{\beta_{2(hx)} - 1\}\{g'(1)\}^2 - 2\sum_{h=1}^{L} \frac{W_h \bar{Y}_h}{nS_{hx}^2}\{B_h \mu_{30(hx)} - \mu_{21(hx)}\} g'(1)$$

Proof: Now mean square error of estimator $\bar{y}_{sg}$ to the first order of approximation is given by

$$MSE(\bar{y}_{sg}) = E(\bar{y}_{sg} - \bar{Y})^2$$

$$= E\left[\sum_{h=1}^{L} W_h \{\bar{Y}_h e_0 - B_h \bar{X}_h e_1 + \bar{Y}_h e_2 g'(1)\}\right]^2$$

(using equation (1.6))

Substituting the values of expectations involved, given in the proof of theorem 1.1, we get

$$MSE(\bar{y}_{sg}) = \sum_{h=1}^{L} W_h \frac{(1-\rho_h^2)}{n} S_{hy}^2 + \sum_{h=1}^{L} \frac{W_h \bar{Y}_h^2}{n}\{\beta_{2(hx)} - 1\}\{g'(1)\}^2 - 2\sum_{h=1}^{L} \frac{W_h \bar{Y}_h}{nS_{hx}^2}\{B_h \mu_{30(hx)} - \mu_{21(hx)}\} g'(1) \qquad (1.9)$$

Theorem 1.3: Optimum class of estimators having minimum mean square error given by,

$$MSE(\bar{y}_{sg})_{\min} = \frac{1}{n}\sum_{h=1}^{L} W_h (1-\rho_h^2) S_{hy}^2 - \frac{1}{n}\sum_{h=1}^{L} W_h \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}^2}{S_{hx}^4 \{\beta_{2(hx)} - 1\}}$$

Satisfies the condition, $g'(1) = \dfrac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}}{\bar{Y}_h S_{hx}^2 \{\beta_{2(hx)} - 1\}}$

Proof: To obtain optimum class of estimators minimizing $MSE(\bar{y}_{sg})$, we proceed as follows:

From equation (8.1.9), we have

$$MSE(\bar{y}_{sg}) = \sum_{h=1}^{L} W_h \frac{(1-\rho_h^2)}{n} S_{hy}^2 + \sum_{h=1}^{L} \frac{W_h \bar{Y}_h^2}{n}\{\beta_{2(hx)} - 1\}\{g'(1)\}^2 - 2\sum_{h=1}^{L} \frac{W_h \bar{Y}_h}{nS_{hx}^2}\{B_h \mu_{30(hx)} - \mu_{21(hx)}\} g'(1)$$

by the principle of Maxima-Minima, partially differentiating $MSE(\bar{y}_{sg})$ with respect to $g'(1)$, the optimum of $g'(1)$ for which $MSE(\bar{y}_{sg})$ is minimum is obtained as

$$g'(1) = \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}}{\bar{Y}_h S_{hx}^2 \{\beta_{2(hx)} - 1\}} = \alpha \text{ (say)} \qquad (1.10)$$

And for this value of $g'(1) = \alpha$, the minimum mean square error of $\bar{y}_{sg}$ is

$$MSE(\bar{y}_{sg})_{\min} = \sum_{h=1}^{L} W_h^2 \frac{(1-\rho_h^2)}{n_h} S_{hy}^2 - \sum_{h=1}^{L} W_h^2 \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}^2}{n_h S_{hx}^4 \{\beta_{2(hx)} - 1\}}$$

Under proportion allocation

$$MSE(\bar{y}_{sg})_{\min} = \frac{1}{n}\sum_{h=1}^{L} W_h (1-\rho_h^2) S_{hy}^2 - \frac{1}{n}\sum_{h=1}^{L} W_h \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}^2}{S_{hx}^4 \{\beta_{2(hx)} - 1\}} \qquad (1.11)$$

Theorem 1.4: $\bar{y}_{sg}$ is more efficient than the conventional estimator $\bar{y}_{ls}$ in the sense of having lesser mean square error under optimum condition,

$$g'(1) = \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}}{\bar{Y}_h S_{hx}^2 \{\beta_{2(hx)} - 1\}}$$

Proof: We know that

$$MSE(\bar{y}_{ls}) = \frac{1}{n}\sum_{h=1}^{L} W_h (1-\rho_h^2) S_{hy}^2$$

Using equation (8.1.11), we see that

$$MSE(\bar{y}_{sg})_{\min} = MSE(\bar{y}_{ls}) - \frac{1}{n}\sum_{h=1}^{L} W_h \frac{\{B_h \mu_{30(hx)} - \mu_{21(hx)}\}^2}{S_{hx}^4 \{\beta_{2(hx)} - 1\}}$$

Which is always greater or equal to zero, showing that the proposed estimator $\bar{y}_{sg}$ has lesser mean square error than $\bar{y}_{ls}$ under optimum condition given by (1.10). Therefore, $\bar{y}_{sg}$ is more efficient than the conventional estimator $\bar{y}_{ls}$ in the sense of having lesser mean square error under optimum condition.

2. Estimated Optimum Class of Estimators

An The optimum value of α in (1.10) or its guessed value may be rarely known in practice, hence it is replaced by its estimate from sample values. Thus, replacing $\mu_{21(hx)}$, $\mu_{30(hx)}$, $\mu_{40(hx)}$, S_{hx}^2 and $\bar{Y}_h$ by their following estimators

$$\hat{\beta}_{2(hx)} = \frac{\hat{\mu}_{40(hx)}}{\hat{\mu}_{20(hx)}^2} \text{ with } \hat{\mu}_{40} = \frac{1}{n_h}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)^4 \ ,$$

$$\hat{\mu}_{20} = s_{hx}^2 = \frac{1}{n_h}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)^2$$

$$\hat{\mu}_{30} = \frac{1}{n_h}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)^3 \quad ,$$

$$\hat{\mu}_{21} = \frac{1}{n_h - 1}\sum_{i=1}^{n_h}\left(x_{hi} - \bar{x}_h\right)^2\left(y_{hi} - \bar{y}_h\right) \quad \text{and}$$

$$\hat{B}_h = b_h$$

We get the estimated optimum value $\hat{\alpha}$ to be

$$\hat{\alpha} = \frac{\left\{b_h \hat{\mu}_{30(hx)} - \hat{\mu}_{21(hx)}\right\}}{\bar{y}_h s_{hx}^2\left\{\hat{\beta}_{2(hx)} - 1\right\}} \tag{2.1}$$

The mean square error in case of estimated optimum $\hat{\alpha}$ is obtained as follows:

From (1.10), we need a function $g(u)$ involves in $\bar{y}_{sg}$ such that $g(1) = 1$, $g(u) = \alpha$

Which means $g(.)$ should involve not only u but α also and thus we need a function $g^*(u,\alpha)$ such that

$$g^*(1,\alpha) = 1 \; , \qquad \left.\frac{\partial g^*}{\partial u}\right]_{(1,\alpha)} = \alpha$$

$$\left.\frac{\partial g^*}{\partial \alpha}\right]_{(1,\alpha)} = 0$$

As the function $g^*(u,\alpha)$ so found involves unknown α, we replace α by its estimate $\hat{\alpha}$ from (2.1) and get the function $g^{**}\left(u,\hat{\alpha}\right)$ such that

$$g^*(1,\alpha) = 1 \; \left.\frac{\partial g^*}{\partial u}\right]_{(1,\alpha)} = \alpha \; \left.\frac{\partial g^*}{\partial \hat{\alpha}}\right]_{(1,\alpha)} = 0 \tag{2.2}$$

Using such a function $g^{**}\left(u,\hat{\alpha}\right)$ satisfying (2.2), we may take

$$\bar{y}_{sg}^{**} = \sum_{h=1}^{L} W_h \bar{y}_{hg}^{**} \tag{2.3}$$

where $\bar{y}_{hg}^{**} = \left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{g^{**}\left(u,\hat{\alpha}\right)\right\}$ as modified estimated optimum class of estimators of Population mean $\bar{Y}$, now expanding $g^{**}\left(u,\hat{\alpha}\right)$ in $\bar{y}_{hg}^{**}$ about the point $P = (1,\alpha)$ in Taylor's series, we have,

$$\bar{y}_{hg}^{**} = \left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{g^{**}\left(1,\alpha\right) + (u-1)\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + \left(\hat{\alpha} - \alpha\right)\left.\frac{\partial g^{**}}{\partial \hat{\alpha}}\right]_{(1,\alpha)} + \ldots\right\}$$

$$= \left[\bar{Y}_h(1+e_0) + B_h(1 - e_2 + e_3 + \ldots)\left\{\bar{X}_h - \bar{X}_h(1+e_1)\right\}\right] * \left\{ \begin{array}{l} g^{**}\left(1,\alpha\right) + e_2\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + \\ \left(\hat{\alpha} - \alpha\right)\left.\frac{\partial g^{**}}{\partial \hat{\alpha}}\right]_{(1,\alpha)} + \ldots \end{array}\right\}$$

$$= \bar{Y}_h(1+e_0) - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + \ldots) + \bar{Y}_h(e_2 + e_0e_2)\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + B_h\bar{X}_h(-e_1e_2 + \ldots)\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + \ldots \tag{2.5}$$

Using (8.2.5) in (8.2.3), we have

$$\bar{y}_{sg}^{**} - \bar{Y} = \sum_{h=1}^{L} W_h\left[\begin{array}{l}\bar{Y}_h e_0 - B_h\bar{X}_h(e_1 - e_1e_2 + e_1e_3 + \ldots) + \bar{Y}_h(e_2 + e_0e_2)\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + \\ B_h\bar{X}_h(-e_1e_2 + \ldots)\left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} + \ldots\end{array}\right] \tag{2.6}$$

Squaring both sides of (2.6), taking terms up to $O\left(\frac{1}{n}\right)$, and taking expectation, the $MSE\left(\bar{y}_{sg}^{**}\right)$ will be

$$MSE\left(\bar{y}_{sg}^{**}\right) = \sum_{h=1}^{L} W_h^2\frac{\left(1-\rho_h^2\right)}{n_h}S_{hy}^2 - \sum_{h=1}^{L} W_h^2\frac{\left\{B_h\mu_{30(hx)} - \mu_{21(hx)}\right\}^2}{n_h S_{hx}^2\left\{\beta_{2(hx)} - 1\right\}}$$

$$MSE\left(\bar{y}_{sg}^{**}\right) = \frac{1}{n}\sum_{h=1}^{L} W_h\left(1-\rho_h^2\right)S_{hy}^2 - \frac{1}{n}\sum_{h=1}^{L} W_h\frac{\left\{B_h\mu_{30(hx)} - \mu_{21(hx)}\right\}^2}{S_{hx}^4\left\{\beta_{2(hx)} - 1\right\}} \tag{2.7}$$

Which equal to the $MSE\left(\bar{y}_{sg}\right)$ given in (1.11), if

$$\left.\frac{\partial g^{**}}{\partial \hat{\alpha}}\right]_{(1,\alpha)} = 0 \tag{2.8}$$

Thus, considering the function $g^{**}\left(u,\hat{\alpha}\right)$ such that

$$g^{**}(1,\alpha) = 1, \; \left.\frac{\partial g^{**}}{\partial u}\right]_{(1,\alpha)} = \alpha \; \text{ and } \; \left.\frac{\partial g^{**}}{\partial \hat{\alpha}}\right]_{(1,\alpha)} = 0 \tag{2.9}$$

(2.9) We get the estimator $\bar{y}_{hg}^{**}$ depending on estimated optimum values as

$$\bar{y}^{**}_{hg} = \left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{g^{**}\left(u, \hat{\alpha}\right)\right\} \qquad (2.10)$$

Which attains the same minimum MSE as given in equation (1.11)

3. Concluding Remarks

I. It may be easily seen that following estimators are special cases of the proposed class of estimators $\bar{y}_{sg}$

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{\left(\frac{S^2_{hx}}{s^2_{hx}}\right)^k\right\}^2\right]$$

where $g(u) = u^{-2k}$, $u = \dfrac{s^2_{hx}}{S^2_{hx}}$

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{1 - \theta\left[\left(\frac{s^2_{hx}}{S^2_{hx}}\right)^k - 1\right]\right\}^2\right],$$

where $g(u) = \left\{1 - \theta\left[\left(\dfrac{s^2_{hx}}{S^2_{hx}}\right)^k - 1\right]\right\}^2$; $u = \dfrac{s^2_{hx}}{S^2_{hx}}$

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{s^2_{hy}\frac{S^2_{hx}}{s^2_{hx}}\right\}^2\right],$$

where $g(u) = \left(\dfrac{s^2_{hx}}{S^2_{hx}}\right)^{-2}$; $u = \dfrac{s^2_{hx}}{S^2_{hx}}$

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{s^2_{hy}\frac{s^2_{hx}}{S^2_{hx}}\right\}^2\right],$$

where $g(u) = u^2$; $u = \dfrac{s^2_{hx}}{S^2_{hx}}$

$$\bar{y}_{sg} = \sum_{h=1}^{L} W_h\left[\left\{\bar{y}_h + b_h\left(\bar{X}_h - \bar{x}_h\right)\right\}\left\{s^2_{hy}\left(\frac{s^2_{hx}}{S^2_{hx}}\right)^k\right\}^2\right],$$

where $g(u) = u^{2k}$; $u = \dfrac{s^2_{hx}}{S^2_{hx}}$

II. Bias of the proposed estimator $\bar{y}_{sg}$ is given as follows:

$$Bias\left(\bar{y}_{sg}\right) = \sum_{h=1}^{L}\frac{W_h B_h \mu_{30(hx)}}{n_h S^2_{hx}}\{1 - g'(1)\} + \sum_{h=1}^{L}\frac{W_h \mu_{21(hx)}}{n_h}\left(\frac{1}{S^2_{hx}} - \frac{B_h}{S_{hxy}}\right) + \sum_{h=1}^{L}\frac{W_h \bar{Y}_h}{2!\,n_h}\left(\beta_{2(hx)} - 1\right)g''(1)$$

III. Mean Square error for the proposed generalized class of separate- regression type estimator under optimum condition is

IV. $$MSE\left(\bar{y}^{**}_{sg}\right) = \frac{1}{n}\sum_{h=1}^{L} W_h\left(1 - \rho^2_h\right)S^2_{hy} - \frac{1}{n}\sum_{h=1}^{L} W_h\frac{\left\{B_h\mu_{30(hx)} - \mu_{21(hx)}\right\}^2}{S^4_{hx}\left\{\beta_{2(hx)} - 1\right\}}$$

It has been shown that a generalized class of $\bar{y}_{sg}$ depending upon estimated optimum value $\hat{g}(1)$, retains the same minimum mean square error given by (8.1.9). Also $g^{**}\left(u, \hat{\alpha}\right)$ solely depends upon sample information and therefore may be preferred to other estimators for more practical utility.

V. From equation (1.9), we have

$$MSE\left(\bar{y}_{sg}\right) = \sum_{h=1}^{L} W_h\frac{\left(1 - \rho^2_h\right)}{n}S^2_{hy} + \sum_{h=1}^{L}\frac{W_h\bar{Y}^2_h}{n}\left\{\beta_{2(hx)} - 1\right\}\left\{g'(1)\right\}^2 - 2\sum_{h=1}^{L}\frac{W_h\bar{Y}_h}{nS^2_{hx}}\left\{B_h\mu_{30(hx)} - \mu_{21(hx)}\right\}g'(1) \qquad (3.1)$$

$$MSE\left(\bar{y}_{ls}\right) = \sum_{h=1}^{L} W_h\frac{\left(1 - \rho^2_h\right)}{n}S^2_{hy} \qquad (3.2)$$

The optimum value of $g'(1)$ for which MSE of $\bar{y}_{sg}$ is minimum is

$$g'(1) = \frac{\left\{B_h\mu_{30(hx)} - \mu_{21(hx)}\right\}}{\bar{Y}_h S^2_{hx}\left\{\beta_{2(hx)} - 1\right\}} = \alpha$$

It is clear from (3.2) that $MSE\left(\bar{y}_{sg}\right) < MSE\left(\bar{y}_{ls}\right)$ if,

$$\left[1 - \frac{2\alpha}{g'(1)}\right] < 0 \qquad (3.3)$$

Now, if $g'(1) > 0$, the efficiency condition (3.3) for $\bar{y}_{sg}$ to be better than $\bar{y}_{ls}$ in the sense of having less mean square error reduces to

$$\alpha > \frac{g'(1)}{2} \qquad (3.4)$$

Further, if $g'(1) > 0$, the efficiency condition (3.3) reduces to

$$\alpha > \frac{g'(1)}{2} \qquad (3.5)$$

This is the situations where we have prior information about the upper or lower bounds or range of α on the basis of the past data, pilot study or experience, we can find better estimators than $\bar{y}_{ls}$ from a class of estimators represented by $\bar{y}_{sg}$ by choosing the function $g(u)$ suitably. If, we know that $\lambda > \alpha_0(> 0)$, we